PERGAMON

International Journal of Heat and Mass Transfer 45 (2002) 3259–3262

International Journal of HEAT and MASS TRANSFER

www.elsevier.com/locate/ijhmt

Technical Note

On the discrete ordinates method for radiative heat transfer in anisotropically scattering media

L.H. Liu *, L.M. Ruan, H.P. Tan

School of Energy Science and Engineering, Harbin Institute of Technology, 92 West Dazhi Street, Harbin 150001, People's Republic of China

Received 9 December 2001

Abstract

In the discrete ordinates method (DOM), the normalized condition for the numerical quadrature of some complex scattering phase functions may not be satisfied. In this paper, a revised discrete ordinates method (RDOM) is developed to overcome this problem, in which a renormalizing factor is added into the numerical quadrature of in-scattering term. The RDOM is used to solve the radiative transfer problem in one-dimensional anisotropically scattering media with complex phase function. The radiative heat fluxes obtained by the RDOM are compared with those obtained by the conventional discrete ordinates method (CDOM) and Monte Carlo method. The results show the RDOM can overcome the false scattering resulted from the numerical quadrature of in-scattering term and improve largely the accuracy of solution of the radiative transfer equation by comparison with the CDOM.

Keywords: Radiative heat transfer; Discrete ordinates method; Anisotropic scattering; Renormalization

1. Introduction

The development of numerical techniques to solve the radiative transfer equation (RTE) has received increased attention since the 1980s. The discrete ordinates method (DOM) [1,2] has emerged as a successful one which satisfies the requirements of compatibility with the numerical schemes used to model the other transfer modes and acceptable computational economy. Chai et al. [3,4] discussed the ray effects and false scattering in DOM and evaluated the three common spatial differencing schemes. Fiveland [5] presented new level symmetric and equal weight quadrature to satisfy some high order moments of scattering phase functions. Ramankutty and Crosbie [6,7] presented a modified discrete ordinate solution of radiative transfer by breaking the radiative intensity into direct and diffuse components, in which the direct component is determined analytically, and the diffuse transport equation is solved numerically by conventional discrete ordinates procedure. Although the efforts of numerical researchers to improve the discrete ordinates solution methodology have led to some interesting innovations in angular quadratures and spatial differencing schemes, the improvement in the DOM to tackle the ray effects and false scattering is needed for the future work.

The general transfer relations are represented by a discrete set of equations for the average intensity over a finite number of ordinate directions in DOM, and the discrete ordinates approximation of RTE is written as

$$\mu_i \frac{\partial I_i}{\partial \tau_x} + \eta_i \frac{\partial I_i}{\partial \tau_y} + \xi_i \frac{\partial I_i}{\partial \tau_z} = (1-\omega) I_b - I_i + \frac{\omega}{4\pi} \times \sum_{j=1}^{M} w_j I_j \Phi_{ij}, \tag{1}$$

where I is the radiative intensity, τ is the optical variable, ω is the scattering albedo, Φ is the scattering phase function, w is the angular weight, μ, η, and ξ are the

* Corresponding author. Tel.: +86-451-641-3230; fax: +86-451-622-1048.

E-mail address: liulh_hit@263.net (L.H. Liu).

0017-9310/02/$ - see front matter
PII: S0017-9310(02)00035-2

Nomenclature

f	*renormalizing factor*
I	radiative intensity
m	complex refractive index
q	radiative heat fluxes
T	temperature
w	angular weight
x	size parameter of particle
μ, η, ξ	direction cosines
τ	optical variable
τ_L	optical thickness
Φ	scattering phase function
ω	scattering albedo
Superscripts	
CDOM	conventional discrete ordinates method
MC	Monte Carlo method
RDOM	revised discrete ordinates method
Subscripts	
i, j	indices for ordinate directions
L	at the boundary of $\tau = \tau_L$
0	at the boundary of $\tau = 0$

direction cosines, and subscripts i and j are the indices for ordinate directions. Theoretically, the scattering phase function is a function normalized such that

$$\frac{1}{4\pi}\int_{4\pi}\Phi(\Omega,\Omega')\,\mathrm{d}\Omega' = 1. \qquad (2)$$

The essential element of the DOM is the quadrature set. Complex scattering phase functions may contain high order moments. At present, all quadrature sets of the DOM only satisfy the integrals of some finite order moments. As an example, Fig. 1 shows the Mie scattering phase function of the particle with complex refractive index $m = 2$ and size parameter $x = 4$. It is a high degree nonlinear phase function. The integral of this scattering phase function by the level symmetric even (LSE) quadrature of DOM S_6 is shown in Table 1. It can be found that the normalized condition for some complex scattering phase functions may not be satisfied, i.e.,

$$\frac{1}{4\pi}\sum_{j=1}^{M} w_j\Phi_{ij} \neq 1. \qquad (3)$$

This will result in false scattering to some extent and affect the accuracy of the DOM, especially in scattering-dominated cases.

A revision is proposed here, with a renormalizing factor, f, added into the integral of in-scattering term, so as to solve the radiative transfer problem in anisotropically scattering media.

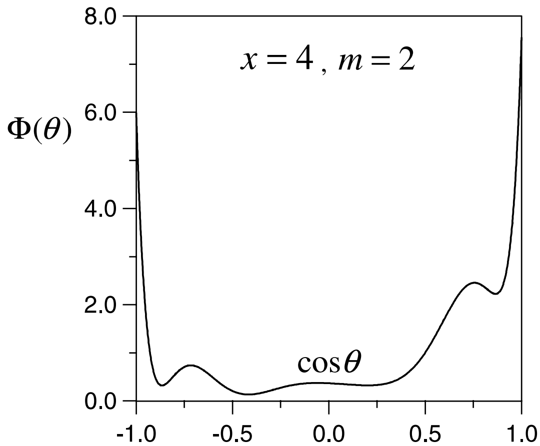

Fig. 1. Mie scattering phase function for the particle with complex refractive index $m = 2$ and size parameter $x = 4$.

2. A revised discrete ordinates method proposed

The integral of in-scattering term is renormalized as the following:

Table 1
The integral of scattering phase function for the discrete directions in the first octant

I	μ_i	η_i	ξ_i	w_i	$(1/4\pi)\sum_{j=1}^{M} w_j\Phi_{ij}$
1	0.9261811	0.2666352	0.2666352	0.2766681	1.0465105
2	0.6815078	0.2666352	0.6815078	0.2469424	0.9923266
3	0.6815078	0.6815078	0.2666352	0.2469424	0.9923266
4	0.2666352	0.2666352	0.9261811	0.2766681	1.0465105
5	0.2666352	0.6815078	0.6815078	0.2469424	0.9923266
6	0.2666352	0.9261811	0.2666352	0.2766681	1.0465105

$$\mu_i \frac{\partial I_i}{\partial \tau_x} + \eta_i \frac{\partial I_i}{\partial \tau_y} + \xi_i \frac{\partial I_i}{\partial \tau_z} = (1-\omega) I_b - I_i + \frac{\omega}{4\pi} \sum_{j=1}^{M} w_j f_j I_j \Phi_{ij}, \quad (4)$$

where f_j is the renormalizing factor defined as

$$f_j = \left(\frac{1}{4\pi} \sum_{i=1}^{M} w_i \Phi_{ji} \right)^{-1}, \quad (5)$$

and is only added into the numerical quadrature of in-scattering term. The treatment of incident energy and heat flux retains as in the conventional DOM.

To validate the accuracy of this revision, consider the radiative heat transfer in a one-dimensional gray, anisotropically scattering media bounded by two black isothermal plates at temperature $T_0 = 3000$ K and $T_L = 1000$ K. The optical thickness of slab is τ_L, and the distribution of slab temperature is linear. We take the scattering phase function shown in Fig. 1 as an example. The radiative heat fluxes at boundaries, q_0 and q_L, are solved by revised DOM (RDOM), the conventional DOM (CDOM) and the Monte Carlo (MC) method, respectively. Tables 2 and 3 show the results, in which the LSE quadrature of DOM S_6 is used. For the sake of comparison and analysis, the relative errors of radiative heat flux are defined as

$$\delta_0^{\mathrm{CDOM}} = 100 \left| \frac{q_0^{\mathrm{MC}} - q_0^{\mathrm{CDOM}}}{q_0^{\mathrm{MC}}} \right|, \quad \delta_L^{\mathrm{CDOM}} = 100 \left| \frac{q_L^{\mathrm{MC}} - q_L^{\mathrm{CDOM}}}{q_L^{\mathrm{MC}}} \right|, \quad (6)$$

Table 2
Radiative heat fluxes and relative errors, δ, at the boundary for $\tau = 0$

τ_L	q_0^{MC} (kW/m^2)	q_0^{CDOM} (kW/m^2)	q_0^{RDOM} (kW/m^2)	δ_0^{CDOM} (%)	δ_0^{RDOM} (%)
$\omega = 0.5$					
1.0	3068.5	3072.0	3095.1	0.11	0.87
3.0	1978.9	1943.5	1985.7	1.79	0.34
5.0	1458.4	1412.4	1464.9	3.15	0.45
7.0	1150.6	1098.6	1157.8	4.52	0.63
$\omega = 0.8$					
1.0	3038.9	2998.8	3054.0	1.32	0.50
3.0	2054.6	1934.1	2053.2	5.86	0.07
5.0	1593.3	1440.8	1594.0	9.57	0.04
7.0	1305.6	1131.0	1306.7	13.37	0.08
$\omega = 1.0$					
1.0	2900.9	2806.7	2909.0	3.25	0.28
3.0	1760.6	1369.2	1746.5	22.23	0.80
5.0	1261.7	509.1	1252.0	59.65	0.77
7.0	988.5	−303.8	975.9	130.73	1.27

Table 3
Radiative heat fluxes and relative errors at the boundary for $\tau = \tau_L$

τ_L	q_L^{MC} (kW/m^2)	q_L^{CDOM} (kW/m^2)	q_L^{RDOM} (kW/m^2)	δ_L^{CDOM} (%)	δ_L^{RDOM} (%)
$\omega = 0.5$					
1.0	2046.8	2063.2	2040.6	0.80	0.30
3.0	642.8	659.2	641.3	2.55	0.23
5.0	274.3	285.3	274.6	4.01	0.11
7.0	147.6	154.8	148.1	4.88	0.34
$\omega = 0.8$					
1.0	2472.0	2530.3	2474.2	2.36	0.09
3.0	1019.4	1087.3	1014.8	6.66	0.45
5.0	501.2	555.1	497.6	10.75	0.72
7.0	280.1	319.9	278.5	14.21	0.57
$\omega = 1.0$					
1.0	2900.9	3014.0	2909.0	3.90	0.28
3.0	1760.6	2057.6	1746.5	16.87	0.80
5.0	1261.7	1822.9	1252.0	44.48	0.77
7.0	988.5	1954.5	975.9	97.72	1.27

Table 4
The comparison of relative errors, δ, for five different quadratures in the case ($\tau_L = 5.0,\ \omega = 0.8$)

Quadrature set of S_6	δ_0^{CDOM} (%)	δ_0^{RDOM} (%)	δ_L^{CDOM} (%)	δ_L^{RDOM} (%)
LSE	9.57	0.04	10.75	0.72
LSO	66.18	3.01	82.82	3.75
LSH	47.79	2.66	56.54	2.87
EWE	5.83	0.01	6.46	0.45
EWO	29.54	1.95	31.72	2.87

$$\delta_0^{RDOM} = 100\left|\frac{q_0^{MC} - q_0^{RDOM}}{q_0^{MC}}\right|,$$
$$\delta_L^{RDOM} = 100\left|\frac{q_L^{MC} - q_L^{RDOM}}{q_L^{MC}}\right|. \quad (7)$$

As shown in Tables 2 and 3, the RDOM results agree with the MC results much better than the CDOM result. For the case of pure scattering ($\omega = 1.0$), theoretically, the radiative heat fluxes at boundaries, q_0 and q_L, are equal to each other. The CDOM cannot predict this phenomenon and will result in physical unreality. By introducing renormalizing factor in the numerical quadrature of in-scattering term, the RDOM overcomes this problem.

To improve the accuracy of DOM, many numerical quadrature sets were presented, for example, level symmetric even (LSE) quadrature, level symmetric odd (LSO) quadrature, level symmetric hybrid (LSH) quadrature, equal weight even (EWE) quadrature, and equal weight odd (EWO) quadrature [5]. Table 4 lists the relative errors of radiative heat flux solved by use of the CDOM and the RDOM with five different quadrature sets in the case $\tau_L = 5.0, \omega = 0.8$. As shown in Table 4, for high degree nonlinear scattering phase function, renormalizing factor in the integral of in-scattering term is needed for all these five sets.

3. Conclusions

A revised discrete ordinates method is proposed for radiative transfer in anisotropically scattering media, with a renormalizing factor added into the numerical quadrature of in-scattering term to improve obviously the accuracy of calculating results as compared with the Monte Carlo method.

Acknowledgements

The supports of this work by Fok Ying Tung Education Foundation (No. 71053), and National Natural Science Foundation of China (No. 50176011) are gratefully acknowledged.

References

[1] W.A. Fiveland, Discrete ordinate methods for radiative heat transfer in isotropically and anisotropically scattering media, ASME J. Heat Transfer 109 (1987) 809–812.
[2] J.S. Truelove, Discrete ordinate solutions of the radiation transfer equation, ASME J. Heat Transfer 109 (1987) 1048–1051.
[3] J.C. Chai, H.S. Lee, S.V. Patankar, Ray effect and false scattering in the discrete ordinates method, Numer. Heat Transfer B 24 (1993) 373–389.
[4] J.C. Chai, S.V. Patankar, H.S. Lee, Evaluation of spatial differencing practices for the discrete ordinates method, J. Thermophys. Heat Transfer 8 (1994) 140–144.
[5] W.A. Fiveland, The selection of discrete ordinate quadrature sets for anisotropic scattering, in: Fundamentals of Radiation Heat Transfer, ASME HTD-vol. 160, 1991, pp. 89–96.
[6] M.A. Ramankutty, A.L. Crosbie, Modified discrete ordinates solution of radiative transfer in two-dimensional rectangular enclosures, J. Quant. Spectrosc. Radiat. Transfer 57 (1997) 107–140.
[7] M.A. Ramankutty, A.L. Crosbie, Modified discrete ordinates solution of radiative transfer in three-dimensional rectangular enclosures, J. Quant. Spectrosc. Radiat. Transfer 60 (1998) 103–134.